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MECHANICS OF BIMODULAR COMPOSITE STRUCTURES

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ABSTRACT

This paper is a survey of the mechanics of beam and plate structures laminated of fiber-reinforced composite materials having different elastic and thermoelastic properties in tension and compression. Examples of such materials include tire cord-rubber, wire-reinforced solid propellants, and soft biological materials. Specific topics covered include: mathematical models of fiber-reinforced bimodular materials and their experimental verification; static and dynamic analysis of laminated and sandwich beams; plane elasticity; analysis of deflection and free and transient vibration of laminated plates and shells. In all of these analyses, thickness-shear deformation and rotatory inertia are included. The solution methods used include closed-form, transfer-matrix, and finite-element techniques.

KEYWORDS

Beams; bimodular materials; fiber-reinforced materials; finite-element method; laminates; plates; sandwich structures; shells; static deflection; thermal stresses; transfer-matrix method; vibration.

INTRODUCTION

The knowledge that certain materials have different stress-strain behavior when they are loaded in tension in contrast to compression goes back more than a century (Saint-Venant, 1864). However, apparently Timoshenko (1941) was the first to consider the bilinear approximation for the uniaxial stress state, with one slope (modulus) in tension and a distinctly different one in compression. Modern interest in the topic stems from the general isotropic formulation of Ambartsumyan (1965), who is credited with originating the "bimodulus" or "bimodular" terminology.

Experimental evidence of drastically different stress-strain behavior in tension and compression has been found in numerous materials, including cast iron (Gilbert, 1961), tire cord-rubber (Clark, 1963), polyamides (Zemlyakov, 1965), polycrystalline graphites (Seldin, 1966), wire-reinforced solid propellants (Herrmann et al., 1967), concrete (Seefried et al., 1967), epoxies (Novak and Bert, 1968), carbon-carbon composites (Adsit et al., 1972), cortical bone (Simkin and Robin, 1973), fiber-reinforced polymers (Jeness and Kline, 1974), rocks (Haimson and Tharp,

1974), compacted clay soils (Ajaz and Parry, 1975), sintered metal (Ducheyne et al., 1978), soft biological tissues (Pearsall and Roberts, 1978), and paperboard (Carlsson et al., 1980); see Table 1. It should be remarked that the no-tension material model often used in rock mechanics (Zienkiewicz et al., 1968) is an extreme limiting case of bimodular material. For an extensive survey of bimodular material behavior as well as the strength-differential phenomenon (different strengths in tension and in compression), the reader is referred to the work of Kamiya (1976a).

TABLE 1 Some Examples of Different Elastic Behavior in Tension and Compression

Investigator	Material	Young's Moduli Ratio E^c/E^t
Kotlarskii & Karbasova (1968)	Fabric/rubber	0.38
Ducheyne et al. (1978)	Sintered, porous stainless steel	0.1
Zolotukhina & Lepetov (1968)	Various fabric/rubber	0.07 to 0.50
Patel et al. (1976)	Polyester cord/rubber	0.017
	Aramid cord/rubber	0.0034
Pósfalvi (1977)	Rayon cord/rubber	0.0036
Pearsall & Roberts (1978)	Myometrium (uterine muscle)	0.2

Here, initial emphasis is placed on the mechanics of fiber-reinforced composites with bimodular action on two different levels: micromechanics to explain the physical phenomena involved and anisotropic elastic continuum theory for use in structural mechanics. This is followed by applications to laminated and sandwich beams, plane elasticity, plate bending, and shells.

MICROMECHANICS OF COMPOSITES WITH CURVED FIBERS AND SOFT MATRICES

To illustrate the severity in bimodular material action, we present stress-strain data in Fig. 1 for unidirectional polyester-cord-reinforced rubber, taken from the work of Bert and Kumar (1981). The basic phenomena taking place are fiber straightening due to tie-bar action in tension and fiber curving due to column action in compression. Both of these phenomena are reacted or restrained by interaction of the fibers with the matrix, which acts like an elastic foundation. The first model based on these phenomena is due to Herrmann et al. (1967). A more recent micromechanics model is due to Cominou (1976). Numerous models of this type were mentioned in a recent paper (Bert, 1979), in which the effects of fiber tension or compression on thermal expansion coefficient were also predicted. The latter are in qualitative agreement with the experimental thermal-expansion results of Makarov and Nikolaev (1971).

Another analytical approach to the micromechanics of composites with curved fibers is the mean-fiber-angle concept originated by Tarnopol'skii et al. (1967). They assumed that the fibers are initially sinusoidally curved with an initial deflection given by

$$w = W_o \sin \beta_o x \qquad (1)$$

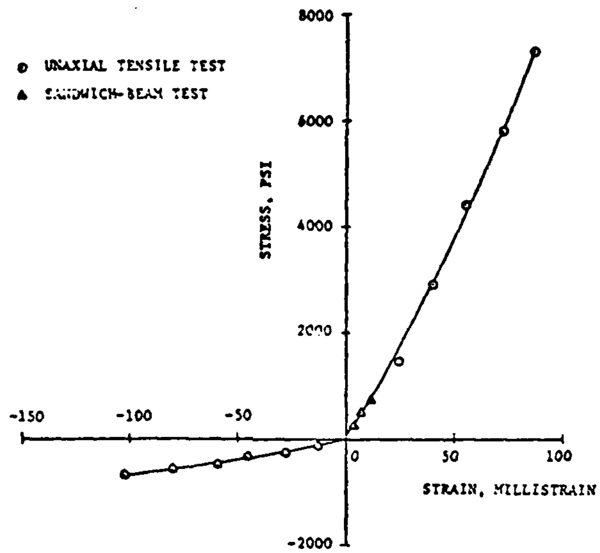

Fig. 1. Stress-strain curve for polyester-rubber in the cord direction (1 psi = 6.895×10^3 pascals).

Then the inclination at an arbitrary axial position x is

$$\theta = \arctan (\beta_o W_o \cos \beta_o x) \tag{2}$$

Using equation (2) in the well-known transformation equations for Young's modulus and integrating over a half-wave length ℓ, they obtained the following result for the fiber-direction initial Young's modulus of a wavy-fiber-reinforced composite:

$$\begin{aligned}(1/\tilde{E}_1) = {} & (1/2E_1)(2 + f_o^2)(1 + f_o^2)^{-3/2} \\ & + (G_{12}^{-1} - 2\nu_{12}E_1^{-1})(f_o^2/2)(1 + f_o^2)^{-3/2} \\ & + (1/2E_2)[2 - (2 + 3f_o^2)(1 + f_o^2)^{-3/2}]\end{aligned} \tag{3}$$

Here E_1 and E_2 are the major (fiber direction) and minor Young's moduli of a unidirectional composite with straight fibers, $\tilde{E}_1$ is the major Young's modulus of the wavy-fiber composite, $f_o \equiv \beta_o W_o$, and G_{12} and ν_{12} are the respective major shear modulus and major Poisson's ratio of the straight-fiber composite.

For $f_o \ll 1$, the following approximate expression was suggested

$$\tilde{E}_1/E_1 \simeq [1 + (f_o^2/2)(E_1/G_{12})]^{-1} \tag{4}$$

The disadvantage of using either equation (3) or (4) is that it requires prior knowledge of a straight-fiber composite having the same fiber volume fraction. For the glass-fiber reinforced plastic investigated experimentally by Tarnopol'skii et al. (1967), it was found empirically that an initial composite prestress of approximately 10% of the ultimate tensile strength was necessary to assure that

the fibers are sufficiently straight. This work can be used to predict only the initial modulus (or Poisson's ratio). However, in certain composites with a highly flexible matrix (such as rubber), the composite modulus may change quite significantly with loading (especially in compression, as discussed earlier in this section). Thus, the analysis of Tabaddor and Chen (1971) is quite important for such composites, since it permits computation of the complete stress-strain curve in both tension and compression for the composite in the fiber direction. In their work, they replaced f_o in equation (3) with f ($\equiv \beta W$) and developed the following relationship between f and f_o, where W is defined in

$$w = W \sin \beta x \tag{5}$$

$$f = \beta W = \beta_o W_o (1 + \epsilon_2)/(1 + \epsilon_1) \tag{6}$$

Later Tarnopol'skii et al. (1973) investigated the effect of fiber waviness in composites with anisotropic fibers (specifically carbon fibers). They found that for such composites, it is necessary to use the full equation (3) rather than the simplified equation (4). Van Dreumel and Kamp (1977) investigated experimentally the effects of fiber waviness on both tensile strength and major Poisson's ratio of carbon-fiber reinforced plastic.

In order to completely characterize a thin sheet material for use in stress analysis and structural design, one needs a complete set of elastic stress-strain relations, for fiber-direction tension and compression (see the next section), not just the Young's moduli. A rational analytical basis for such a complete characterization was developed by Bert (1979). His expressions for the curved-fiber composite compliances $\tilde{S}_{ij}$ in terms of the straight-fiber unidirectional compliances S_{ij} are as follows:

$$\begin{Bmatrix} \tilde{S}_{11} \\ \tilde{S}_{12} \\ \tilde{S}_{22} \\ \tilde{S}_{66} \end{Bmatrix} = \begin{bmatrix} S_{22} & I_1 & I_2 \\ S_{12} & I_2 & -I_1 \\ S_{11} & I_3 & I_2 \\ S_{66} & I_4 & -I_4 \end{bmatrix} \begin{Bmatrix} 1 \\ \overline{m^2} \\ \overline{m^4} \end{Bmatrix} \tag{7}$$

Here the invariants are

$$\begin{aligned} I_1 &\equiv 2S_{12} - 2S_{22} + S_{66} \quad , \quad I_2 \equiv S_{11} - 2S_{12} + S_{22} - S_{66} \\ I_3 &\equiv - 2S_{11} + 2S_{12} + S_{66} \quad , \quad I_4 \equiv 4S_{11} - 8S_{12} + 4S_{22} - 4S_{66} \end{aligned} \tag{8}$$

The average values of the functions of m ($\equiv \cos \theta$) and n ($\equiv \sin \theta$) appearing in equations (7) are

$$\begin{aligned} \overline{m^4} &\equiv [2 \int_{-L/4}^{L/4} m^4 \, dx]/[2 \int_{-L/4}^{L/4} dx] = [1 + (f^2/2)](1 + f^2)^{-3/2} \\ \overline{m^2} &\equiv (1 + f^2)^{-1/2} \quad , \quad \overline{m^3 n} = \overline{mn^3} = 0 \end{aligned} \tag{9}$$

It is noted that since $\overline{m^3 n} = \overline{mn^3} = 0$, the wavy-fiber composite remains orthotropic. Use of equations (7)-(8) in an iterative manner permits development of complete nonlinear strain versus stress relationships for the composite.

The experiments of Patel et al. (1976) showed that the difference between tension and compression properties were much more pronounced for rubber reinforced with aramid cord than when reinforced with polyester cord. This is probably related to the very low compressive strength of the aramid fibers themselves, which Greenwood and Rose (1974) suggested is due to separation of the microfibrils comprising the fibers. Moncunill de Ferran and Harris (1970) found experimentally that the buckling mode of steel wire/polyester was helical rather than planar as assumed in all known analyses. It has been suggested that the more pronounced difference between tensile and compressive behavior exhibited in the tire-cord/rubber composites of Patel et al. (1976) and Pósfalvi (1977) as compared to that of straight wire-reinforced rubber may be due to the inherent twisting action present in tire cord, which has a helical geometry.

MACROMECHANICS OF BIMODULAR ANISOTROPIC CONSTITUTIVE RELATIONS

The first work on the mechanics of bimodular orthotropic media is due to Ambartsumyan (1965, 1969). Other early models are due to Isabekyan and Khachatryan (1969) and Shapiro (1966). More recent work in this area intended especially for transversely isotropic ATJ-S graphite, is due to Jones and Nelson (1976), Jones (1977a,b), and Jones and Morgan (1977).

For fiber-reinforced composites with very soft matrices (e.g., cord-rubber), Bert (1977) introduced the fiber-governed, symmetric compliance model and applied it very successfully to experimental data reported by Patel et al. (1976). In this model, there are two sets of elastic coefficients: one when the fiber-direction strain (ε_f) is tensile and another when ε_f is compressive. Within each set (tensile and compressive), the compliance matrix is symmetric, so that the concept of strain energy is retained. Thus, within this context, the well-known reciprocal relation relating Young's moduli (E_I) and Poisson's ratios (ν_{IJ}) holds:

$$\nu_{LT}^k/E_L^k = \nu_{TL}^k/E_T^k \qquad (k=c,t) \tag{10}$$

Here, subscripts L and T refer to the directions respectively parallel with and perpendicular to the fiber direction, and superscripts c and t refer to fiber-direction compression or tension strain. It should be emphasized that E_T^c and ν_{TL}^c, although referred to as compressive properties according to the convention mentioned, are actually obtained under uniaxial tensile loading perpendicular to the fibers. In particular, the four properties obtained from uniaxial tensile tests, namely E_L^t, ν_{LT}^t, E_T^c, and ν_{TL}^c, are not related.

A linear material model can be characterized by either its compliance matrix or alternatively by its stiffness matrix. The criteria used to evaluate the various bimodular material models are as follows:

1. The model should be related to the governing physical mechanisms involved, i.e., fiber tie-bar effect in tension and fiber buckling instability in compression.
2. It is highly desirable to have the strain energy be positive definite so that energy is conserved.
3. The compliances should be consistent with measured values for the conditions specified, i.e., they should depend upon the nature of the multiaxial stress or strain state as appropriate.
4. The shear modulus for an orthotropic bimodular material should have different values for shear stresses of same magnitude but opposite sign in any

coordinates other than the material-symmetry directions.

As pointed out by Voigt (1928), criterion 2 implies that: (a) the compliance or stiffness matrix be symmetric and (b) certain limits exist on the compliances or stiffnesses so that the compliance or stiffness matrix is positive definite. Symmetry of the compliance or stiffness matrix is necessary in order for a material to be mechanically stable, as shown by Brun (1976). Furthermore, compliance symmetry is highly desirable in that most structural analysis algorithms are based on this assumption, i.e., they have no provision for unsymmetric compliance or stiffness matrices.

A detailed discussion and critique of the various mathematical models for bimodular anisotropic elastic materials was presented by Bert (1978). Table 2 is a brief summary of the results of this evaluation on the basis of the four criteria mentioned previously.

TABLE 2. Evaluation of Various Macroscopic Bimodular-Material Models

Model	Name	Criterion			
Model	Name	1	2	3	4
I	Ambartsumyan (1965, 1969)	No	No	No	Yes
II	Isabekyan-Khachatryan (1969) restricted compliance	No	Yes	No	No
III	Shapiro (1966) first invariant	No	Yes	No	No
IV	Jones (1977a) weighted compliance	No	Yes	No	Yes
V	Bert (1977) symmetric compliance	Yes	Yes	Yes	Yes

MECHANICS OF BIMODULAR COMPOSITE-MATERIAL, LAMINATED, AND SANDWICH BEAMS

A fairly extensive survey of the literature of bimodular beams was included in the recent work of Tran and Bert (1982). In the present work, however, attention is restricted to composite beams. Many of these were related to the experimental flexural behavior of specific composite materials. Vierling and Scheele (1961) were concerned with unidirectional cord-reinforced rubber and Zolotukhina and Lepetov (1968) with fabric-reinforced rubber. Bilida (1969), Nachlinger and Leininger (1969), Jones (1976), Topper et al. (1978), Zweben (1978), and Phan-Thien (1981) were involved with fiber-reinforced plastics, Carlsson et al. (1980) with paperboard, and Swift and Smith (1979) with reinforced concrete. It should be mentioned that Zweben (1978) and Swift and Smith (1979) assumed that the materials were linearly elastic in tension and perfectly plastic in compression. The effects of transverse shear deformation were included by Tabaddor (1976) and Tran and Bert (1982) for static loading and by Bert and Tran (1982) for transient loading.

Relatively little attention has been devoted to the analysis of beams laminated of bimodular material. Kotlyarskii and Karbasova (1968) were concerned with a multilayer conveyor belt, Crawford (1968) with a boron-polymer laminate, and Jones and Morgan (1980) with laminates of composite materials. Recently, Bert and Clifford Rebello (1982) added the effect of transverse shear deformation and made a detailed investigation of the effects of stacking sequence.

The first investigation of bimodular sandwich beams is due to Bert (1982), who developed equations for the reduction of experimental data on sandwich beams of bimodular material for determination of separate stress-strain curves in tension and in compression for the facing material. This technique has a practical

experimental advantage over the use of compact-section beams (Nadai, 1950) in that it is not necessary to measure the beam deflection. This technique was used to obtain 0° compression loading in the work of Bert and Kumar (1981), see Fig. 1 of the present paper.

Apparently the only investigation of the dynamic behavior of sandwich beams with bimodular facings is due to Bert and Clarence Rebello (1982). They presented both classical (fourth-order bending differential equation) and sixth-order-theory analyses of pin-ended beams and compared the results with experimental results obtained for beams with polyurethane-foam cores and aramid cord-rubber facings.

ORTHOTROPIC, BIMODULAR PLANE ELASTICITY

Apparently the first formulation of the plane problem of elasticity for a bimodular, rectilinearly orthotropic material is due to Ambartsumyan (1969). However, he did not present any solutions. For a bimodular, cylindrically orthotropic material, Kamiya (1977) formulated and solved the plane-strain thermoelastic problem for the axisymmetric case. Later Kamiya (1979) generalized this work to include the effects of temperature-dependent properties.

Tabaddor (1981) formulated plane elasticity problems for rectilinearly orthotropic materials by the finite-element approach and solved the case of a point-loaded, clamped-clamped beam treated as a plane-stress problem.

ORTHOTROPIC AND LAMINATED PLATES UNDERGOING BENDING

Apparently the first formulation and solution of a plate-bending problem for orthotropic bimodular material was due to Bert and Kincannon (1979). They used Bert's fiber-governed symmetric material model (1977) and pointed out the importance of the neutral-surface position associated with vanishing total strain in the fiber direction. The particular problem that they solved was a specially orthotropic, thin plate having an elliptic planform, a uniformly distributed pressure loading, and clamped edge.

The first solution of a laminated bimodular plate of finite dimensions was due to Kincannon et al. (1980). They considered the two-layer, cross-ply version of the aforementioned orthotropic elliptic plate problem. In a cross-ply laminate, the location of the neutral surfaces associated with each of the two fiber directions is important. This is illustrated in Fig. 2, taken from Bert et al. (1981a). In this figure, the neutral-surface position (z_{nx}) associated with the bottom layer (fibers in the x direction) is positive (below the midplane). Thus, compressive properties must be used for the upper portion ($0 \leq z \leq z_{nx}$) of this layer and tensile properties for the bottom portion ($z_{nx} \leq z \leq h/2$, where $h \equiv$ total thickness of the laminate). The neutral-surface position (z_{ny}) associated with vanishing total strain in the y direction governs the top layer and is shown negative, thus falling within the top layer and dividing it into two regions. In summary, in this most general case, bimodular action in this two-layer laminate causes it to act as if it were a four-layer laminate. Of course, it must be remembered that the four "layers" have different thicknesses.

Due to their low thickness shear moduli relative to their Young's moduli, plates constructed of composite materials exhibit significantly larger thickness shear deformations than do isotropic plates of the same geometry and loading. Thus, a plate which may reasonably be considered "thin" (say, with a thickness/side ratio less than 0.1) for isotropic materials may have to be considered "thick" for

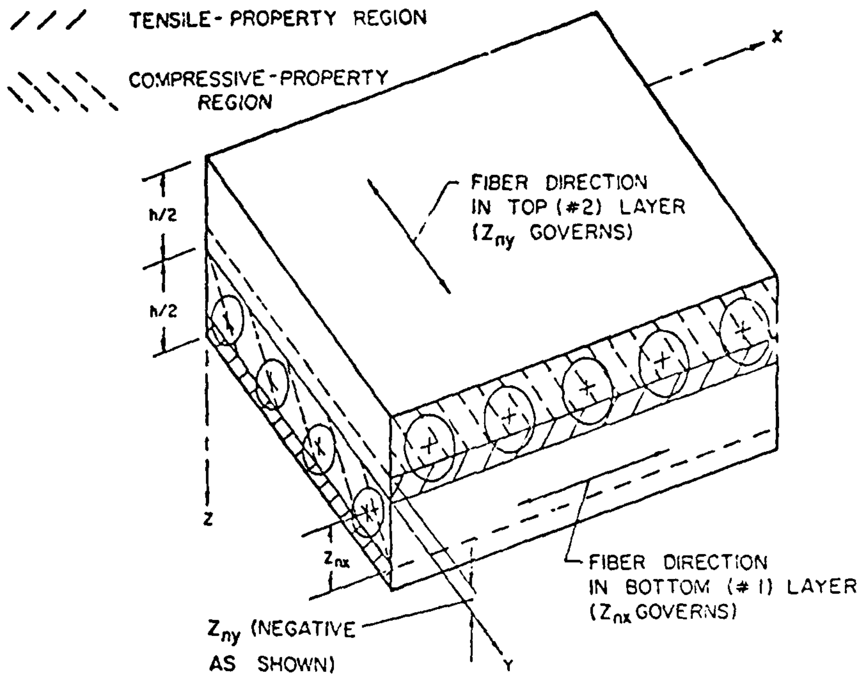

Fig. 2. Schematic diagram of an element of a two-layer, cross-ply plate constructed of bimodular materials showing z_{nx} and z_{ny}, the neutral-surface positions associated with $\epsilon_x = 0$ and $\epsilon_y = 0$. From Bert et al. (1981a).

composite materials. The first analysis of composite-material plates including thickness shear deformation was conducted by Reddy and Bert (1979). They used the finite-element method to analyze both single-layer, specially orthotropic and two-layer, cross-ply plates of rectangular planform. The loading was sinusoidally distributed pressure and the boundary conditions freely supported (simply supported flexurally with no in-plane normal restraint) on all four edges. Closed-form solutions of this problem were later presented for thin plates and for plates with thickness shear deformation (Bert et al., 1980, 1981a). Agreement with the finite-element results was excellent, thus providing a validation of the finite-element solution.

The thermal-stress problem of plates subjected to a change in both midplane temperature and temperature gradient through the thickness was analyzed by Reddy et al. (1980). They presented both closed-form and finite-element results for rectangular plates with thickness shear and subjected to sinusoidal distributions of midplane temperature and temperature gradient.

It is important to mention here that in the case of fiber-reinforced bimodular material, the stretching stiffnesses (A_{ij}), bending-stretching coupling stiffnesses (B_{ij}), and bending stiffnesses (D_{ij}) of the plate all depend upon the "governing" neutral-surface position. Thus, it is possible to obtain closed-form solutions only for problems in which the governing neutral-surface positions remain constant

throughout the plate. In the case of a rectangular planform, this requires sinusoidal distributions of midplane temperature, temperature gradient, and pressure. It was shown by Bert et al. (1981a) that superposition is not valid for bimodular plates, so that the usual Fourier-series approach so popular in plate analysis (Timoshenko and Woinowsky-Krieger, 1959) is not valid. Fortunately, however, the finite-element formulation is not subject to this limitation, i.e., the neutral-surface positions may vary from point to point (x,y position) as required. Thus, Reddy and Chao (1980) considered rectangular plates for the more practical case of uniformly distributed pressure loading, and Reddy et al. (1981) uniform distributions of midplane temperature and temperature gradient. Reddy and Bert (1982) presented numerical results for clamped rectangular plates subjected to uniformly distributed pressure or to point loading at the center.

The first dynamic analyses of bimodular composite-material plates were due to Bert et al. (1981b). They presented both closed-form and finite-element analyses of rectangular plates undergoing free vibration. In the case of a single-layer plate, the plate exhibits the same stiffnesses regardless of the sign of the bending curvature in the fiber direction. Thus, both the positive-curvature portion and the negative-curvature portion of a cycle take place in the same amount of time. However, in the case of a two-layer laminate, due to its antisymmetric nature, the plate stiffnesses are different in the two portions of a cycle and the two portions of a cycle take place in different lengths of time. Since the two time intervals must add up to the total period of one complete cycle, the frequency (ω) defined as 2π/period is found to be related to the frequencies (ω_1, ω_2) of the two portions of a cycle by the following simple relationship:

$$\omega^{-1} = (1/2)(\omega_1{}^{-1} + \omega_2{}^{-1}) \tag{11}$$

Furthermore, Bert et al. (1981b) demonstrated that energy is conserved as a plate goes from the first portion of a cycle to the second portion.

Recently, Reddy (1981) presented both closed-form and finite-element analyses of the transient response of rectangular plates to a suddenly applied, sinusoidally distributed pressure loading. The linear matrix differential equations in time were solved by the well-known beta method due to Newmark (1959). As before, excellent agreement was achieved between the two different types of analyses.

All of the analyses mentioned so far have been linear, although piecewise linear (bimodular). It is believed that the first nonlinear analysis of bimodular composite-material plates is the recent work of Turvey (1981). He considered the large static deflections (geometric nonlinearity) of cross-ply, rectangular, thin plates using the dynamic relaxation method of solution. However, the numerical results that he presented were for material that was only slightly bimodular and slightly orthotropic ($E_L^t/E_L^c = E_L^t/E_T^t = 2$, vs. $E_L^t/E_L^c \doteq 9$ to 20 and $E_L^t/E_T^t \doteq 100$ for aramid-rubber, Bert and Kumar, 1981). Furthermore, he did not explicitly state which criterion for bimodularity (material model) he used. Recently, Reddy and Chao (1982) independently considered the same problem, but with thickness shear deformation included. They used the finite-element method of solution. Some typical results are shown in Fig. 3.

ORTHOTROPIC AND LAMINATED SHELLS

The first formulation and solution of a shell problem for orthotropic bimodular material was due to Jones (1971). He considered buckling of laminated, circular cylindrical thin shells with eccentrically located stiffeners and subjected to arbitrary combinations of axial and circumferential normal loading. He used Donnell-type shell theory and considered the stiffeners to be smeared. Results

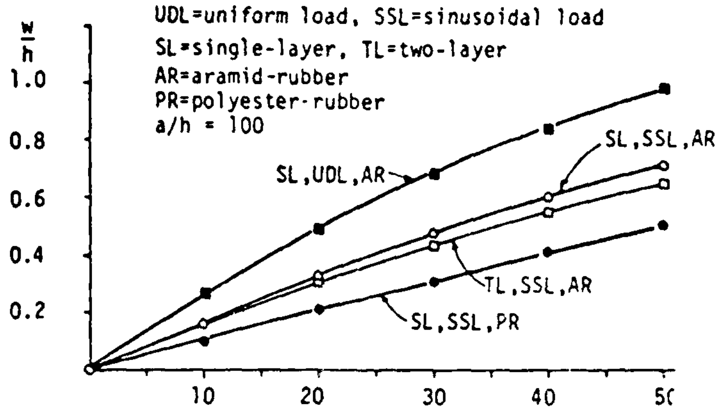

Fig. 3. Load-deflection curves for thin square plates bimodular materials (a/h = 100).

were presented in graphical form for a ring-stiffened, two-layer, c ly shell. Due to the material model used (Ambartsumyan, 1969), there were cert... discontinuities on the buckling diagrams. This work was followed by that of Jones and Morgan (1975a), who presented results for single-layer and two-layer, cross-ply unstiffened shells. In this work they used two different material models, the Jones (1977a) weighted-compliance model and the Isabekyan and Khachatryan (1969) restricted-compliance model. Jones and Morgan (1975b) also compared the results of their analyses to the experimental results obtained by Lenoe et al. (1969) on three-dimensionally reinforced carbon-carbon cylinders.

Static deflection of thin shells of orthotropic bimodular material was analyzed by Kamiya (1976b). He considered single-layer circular cylindrical shells subjected to axisymmetric loading. Both cylindrically curved panels and complete circular-cylindrical shells of single-layer and two-layer, cross-ply constructed were analyzed by Bert and Reddy (1980).

The first treatment of orthotropic bimodular shells including thickness shear deformation was due to Hsu et al. (1981). They treated circular cylindrical shells under both pressure loading and temperature variations. They introduced a generalized theory of thickness-shear-deformation shell theory (TSDST) which, by means of tracer coefficients, reduces to the TSDST versions of the Sanders (1959), Love first-approximation (1927), and Donnell (1933) thin shell theories. For selected combinations of loading and boundary conditions, both closed-form and finite-element solutions were presented for both single-layer and two-layer, cross-ply constructions. See also Reddy et al. (1981).

The first dynamic analysis of bimodular shells was due to Bert and Kumar (1982), who analyzed the free vibration of both thin and thick circular cylindrical shells of cylindrical panel and complete shell configurations.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

It is concluded that many materials, especially fiber-reinforced composite materials, have different stress-strain behavior in tension and compression. Through the bimodular approximation, anisotropic constitutive relations are

available and have been successfully applied to a variety of structural members (especially beams, plates, and shells) by means of both classical and finite-element methods.

There appears to be considerable need for further research in this area, mainly:

1. Development of smooth nonlinear (rather than bimodular) constitutive relations for materials having drastically different behavior in tension and compression. For analysis purposes, these can be treated as multimodular, i.e., piecewise linear. Work on this is currently in progress (Bert and Gordaninejad, 1982).

2. Research on the buckling of both columns and plates constructed of bimodular composite materials with significant transverse shear flexibility.

3. Analysis of plane elasticity problems such as stress concentration at holes, fracture problems, etc.

4. Analysis of nonlinear transient behavior of beams, plates, and shells constructed of bimodular composite materials.

5. Analysis of edge effects at free straight side edges and cutouts in laminates of bimodular composite materials.

6. Analysis of infinite media of anisotropic bimodular materials (an example is the recent wave propagation analysis by Benveniste (1982).

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PREVIOUS REPORTS ON THIS CONTRACT

Project Rept. No.	Issuing University Rept. No.*	Report Title	Author(s)
1	OU 79-7	Mathematical Modeling and Micromechanics of Fiber Reinforced Bimodulus Composite Material	C.W. Bert
2	OU 79-8	Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials	J.N. Reddy & C.W. Bert
3	OU 79-9	Finite-Element Analyses of Laminated Composite-Material Plates	J.N. Reddy
4A	OU 79-10A	Analyses of Laminated Bimodulus Composite-Material Plates	C.W. Bert
5	OU 79-11	Recent Research in Composite and Sandwich Plate Dynamics	C.W. Bert
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16	OU 80-9	Thermal Bending of Thick Rectangular Plates of Bimodulus Material	J.N. Reddy, C.W. Bert, Y.S. Hsu, & V.S. Reddy
17	OU 80-14	Thermoelasticity of Circular Cylindrical Shells Laminated of Bimodulus Composite Materials	Y.S. Hsu, J.N. Reddy, & C.W. Bert
18	OU 80-17	Composite Materials: A Survey of the Damping Capacity of Fiber-Reinforced Composites	C.W. Bert
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20	VPI 81-11 & OU 81-1	On the Behavior of Plates Laminated of Bimodulus Composite Materials	J.N. Reddy & C.W. Bert
21	VPI 81-12	Analysis of Layered Composite Plates Accounting for Large Deflections and Transverse Shear Strains	J.N. Reddy
22	OU 81-7	Static and Dynamic Analyses of Thick Beams of Bimodular Materials	C.W. Bert & A.D. Tran
23	OU 81-8	Experimental Investigation of the Mechanical Behavior of Cord-Rubber Materials	C.W. Bert & M. Kumar
24	VPI 81.28	Transient Response of Laminated, Bimodular-Material Composite Rectangular Plates	J.N. Reddy
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18. SUPPLEMENTARY NOTES

This paper is to be presented at the International Union of Theoretical and Applied Mechanics Symposium on Mechanics of Composite Materials, to be held at the Virginia Polytechnic Institute and State University, Blacksburg, VA, Aug. 16-19, 1982. The proceedings of the symposium are to be published (over)

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Beams, bending-stretching coupling, bimodular materials, buckling, composite materials, cord-rubber composites, elasticity theory, experimental characterization, fiber-reinforced materials, finite-element method, free vibration, geometric nonlinearity, laminates, natural frequencies, nonlinear vibration, plane elasticity, plate bending, sandwich structures, shells, (over)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is a survey of the mechanics of beam and plate structures laminated of fiber-reinforced composite materials having different elastic and thermoelastic properties in tension and compression. Examples of such materials include tire cord-rubber, wire-reinforced solid propellants, and soft biological materials. Specific topics covered include: mathematical models of fiber-reinforced bimodular materials and their experimental verification; static and dynamic analysis of laminated and sandwich beams; plane (over)

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elasticity; analysis of deflection and free and transient vibration of laminated plates and shells. In all of these analyses, thickness-shear deformation and rotatory inertia are included. The solution methods used include closed-form, transfer-matrix, and finite-element techniques.

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